

# Quantum Properties of a Nanomechanical Oscillator

Aziz Kolkiran and G. S. Agarwal

*Department of Physics, Oklahoma State University, Stillwater, OK - 74078, USA*

(Dated: submitted to **Phys. Rev. B** on August 28, 2006)

We study the quantum properties of a nanomechanical oscillator via the squeezing of the oscillator amplitude. The static longitudinal compressive force  $F_0$  close to a critical value at the Euler buckling instability leads to an anharmonic term in the Hamiltonian and thus the squeezing properties of the nanomechanical oscillator are to be obtained from the Hamiltonian of the form  $H = a^\dagger a + \beta(a^\dagger + a)^4/4$ . This Hamiltonian has no exact solution unlike the other known models of nonlinear interactions of the forms  $a^\dagger a^2$ ,  $(a^\dagger a)^2$  and  $a^\dagger + a^4 - (a^\dagger a^2 + a^2 a^\dagger)$  previously employed in quantum optics to study squeezing. Here we solve the Schrödinger equation numerically and show that in-phase quadrature gets squeezed for both ground state and coherent states. The squeezing can be controlled by bringing  $F_0$  close to or far from the critical value  $F_c$ . We further study the effect of the transverse driving force on the squeezing in nanomechanical oscillator.

PACS numbers: 62.25.+g, 42.50.Dv, 05.45.-a, 62.30.+d

## I. INTRODUCTION

There is currently a wide effort to observe quantum behavior in nanoscale devices [1, 2, 3, 4]. In the limit of high resonator frequency with high mechanical quality factors and long coherence lifetimes, the nanomechanical oscillator (NMO) phonons will be analogous to photons in an electromagnetic cavity. With current technology it is possible to reach resonator frequency of GHz order [5]. At a temperature of around 50mK, one can principally prepare the resonator into the ground state. These sub-Kelvin temperatures are well within the range of today's dilution refrigerators. However cooling the resonator down to these temperatures requires some other techniques [6, 7].

With the assumption that quantum mechanics should apply to these mesoscopic systems, variety of methods and techniques have been proposed to observe some quantum optical effects including solid-state laser cooling [6, 7], quantum nondemolition measurement [8, 9], phonon lasing [10] and squeezed state generation [11, 12]. There are also proposals analyzing macroscopic quan-

tum tunneling [13], resonant multi-phonon excitations [14] and a variety of methods to entangle mechanical resonators with other quantum systems [15, 16, 17, 18].

The next question is what could be the best way to study quantum properties of a NMO. In line with the work in quantum optics on squeezing, we can consider studying the squeezed states of the NMO. One proposal considers modulating the spring constant to produce squeezing [11] as it is known from the earlier work [19] that any modulation of the frequency of the oscillator can result in squeezing. Here we adopt a different model. We consider the situation shown schematically in the Fig. 1. We show various forces acting on a nanobeam structure which is clamped at both ends vibrating in the transverse direction. There is a static mechanical force,  $F_0$ , acting in the longitudinal direction and an ac-driving force to excite vibrations in the transverse direction. The longitudinal force  $F_0$  which is close to a critical value at the Euler instability gives rise to an additional term in the potential energy which is quartic in the fundamental mode amplitude  $x$ . The effective Hamiltonian that describes the system would be in the form  $H = p^2/2m + m\omega^2 x^2/2 + \beta x^4/4$ . The derivation of this nonlinearity in  $x$  is given in the next section. Unlike the previous work on squeezing [11] in a NMO we would consider the effect of the nonlinearity in  $x$ . Note that the nonlinearity can be switched on and off by controlling  $F_0$ . We would thus study the quantized behavior of a NMO subject to the force  $F_0$ .

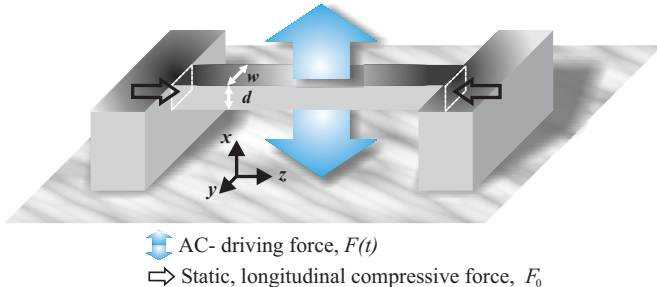


FIG. 1: (Color online) Schematic diagram of the freely suspended nanomechanical beam of total length  $L$ , width  $w$  and thickness  $d$ . The beam is clamped at both ends. A static, mechanical force  $F_0$  compresses the beam in longitudinal direction controlling the nonlinearity. An ac-driving force can be used to excite the beam to transverse vibrations.

The organization of the paper is as follows. The model is described in section II and the effective Hamiltonian is derived briefly by referring to the previous works for the doubly clamped elastic rectangular beam. We discuss the previous works on the squeezing in nonlinear oscillators in section III. Then, we study the quantum dynamics and analyze the squeezing properties in section IV. The conclusion and future perspective are given in section V.

## II. THE MODEL

We start with an elastic rectangular beam of length  $L$ , width  $w$  and thickness  $d$  as shown in Fig. 1. The beam is freely suspended and clamped at both ends. The transverse motion in the direction of  $d$  is allowed. The dimensions are such that ( $L \gg w > d$ ) there is no appreciable vibrations in other directions. A static mechanical force  $F_0$  acts on the beam in the longitudinal

direction ( $F_0 > 0$  for compression). An ac-driving field,  $F(t) = \tilde{f} \cos(\omega_{ex}t)$  can also be added to excite the vibrations. The dynamics of the beam can be completely described by the transverse deflection  $\phi(s)$  parametrized by the arclength  $s \in [0, L]$  in a classical picture. Assuming single transverse degree of freedom for simplicity the nonlinear Lagrangian of the system, for arbitrary strong deflections  $\phi(s)$  is then [20, 21],

$$\mathcal{L}(\phi, \dot{\phi}, t) = \int_0^L ds \left[ \frac{\rho}{2} \dot{\phi}^2 - \frac{\mu}{2} \frac{\phi'^2}{(1 - \phi'^2)} - F_0(\sqrt{1 - \phi'^2} - 1) + F(t)\phi \right]. \quad (1)$$

Here  $\rho = m/L$  is the mass density,  $\mu = EI$  is the product of the elasticity modulus  $E$  and the moment of inertia  $I$ . In  $\phi'$ , prime denotes partial derivative with respect to  $s$ , i.e.  $\partial\phi/\partial s$ . For small oscillations  $|\phi'(s)| \ll 1$ , the Lagrangian is quadratic and it leads to the linear equation of motion

$$\rho \ddot{\phi} + \mu \phi'''' + F_0 \phi'' = 0. \quad (2)$$

The equation of motion can be separated and transformed into an eigenvalue problem with boundary conditions applied to the endpoints. One can write the general solution as a superposition  $\phi(s, t) = \sum_n \phi_n(s, t) = \sum_n \mathcal{A}_n(t) g_n(s)$ , where  $g_n(s)$  are the normal modes which follow as solution of the characteristic equation and  $n = 1$  is called the fundamental mode. For the doubly clamped nanobeam, we have  $\phi(0) = \phi(L) = 0$  and  $\phi'(0) = \phi'(L) = 0$ . The expressions for  $\phi_n$ 's are then given by a superposition of trigonometric and hyperbolic functions, and the eigenfrequencies  $\omega_n$ 's are the solutions of transcendental equations. For the doubly clamped boundary conditions, when  $F_0$  is close to the critical value  $F_c = \mu(\pi/L)^2$ , one can get the simplified fundamental frequency  $\omega_1(F_0 \rightarrow F_c) = \sqrt{\epsilon} \omega_0$  where  $\omega_0 = (2\pi^2/3)\sqrt{E/\rho}(d/L^2)$  is the fundamental frequency of the relaxed beam ( $F_0 = 0$ ). The parameter  $\epsilon = (F_c - F_0)/F_c$  is called the distance to the critical force and the system reaches to the well known Euler instability when  $\epsilon \ll 1$ . The dynamics at low energies close to the Euler instability will be dominated by the fundamental mode alone. The fundamental mode  $g_1(s)$  can also be expanded close to the Euler instability in zeroth order in  $\epsilon$

$$g_1(s) \simeq \sin^2\left(\frac{\pi s}{L}\right). \quad (3)$$

Since the fundamental frequency  $\omega_1$  vanishes at the critical value  $F_c$ , one has to include the contributions beyond the quadratic terms  $\phi'^2$  and  $\phi''^2$  in the Lagrangian. The

next higher order terms,  $-(\mu/2)\phi''^2\phi'^2$  and  $(F_0/4)\phi'^4$ , are quartic in the Lagrangian. Inserting the normal mode expansion  $\phi(s, t) = \sum_n \mathcal{A}_n(t) g_n(s)$  in the Lagrangian and assuming that the fundamental mode  $n = 1$  dominates the dynamics (by neglecting the higher modes  $n = 2, 3, \dots$ ) one can quantize the theory by introducing the canonically conjugate momentum  $p \equiv -i\hbar\partial/\partial\mathcal{A}_1$  with the ‘‘coordinate’’  $x \equiv \mathcal{A}_1$ . Note that when the driving frequency is close to the fundamental frequency of the beam, the fundamental mode will dominate also in absence of a static longitudinal compression force  $F_0$ . However, a compression force close to a critical value helps to enhance the nonlinear effects which are of the importance of this paper.

By using the above definition of coordinate and the conjugate momentum, an effective quantum mechanical time-dependent Hamiltonian results describing the dynamics of a single quantum particle

$$\tilde{H}(t) = \frac{p^2}{2m^*} + \frac{m^* \omega_1^2}{2} x^2 + \frac{\tilde{\beta}}{4} x^4 + xF(t), \quad (4)$$

with the effective mass  $m^* = 3\rho L/8$ , the fundamental frequency  $\omega_1 = \sqrt{\epsilon}(2\pi^2/3)\sqrt{E/\rho}(d/L^2)$  and the nonlinearity parameter  $\tilde{\beta} = (\pi/L)^4 F_c L(1 + 3\epsilon)$  [22]. Now, Eq. (4) can be put in a second-quantized form by replacing  $x$  and  $p$  with the creation and annihilation operators  $a^\dagger$  and  $a$ ,

$$x = \sqrt{\frac{\hbar}{m^* \omega_1}} \frac{1}{\sqrt{2}} (a^\dagger + a) \quad (5)$$

$$p = \sqrt{m^* \hbar \omega_1} \frac{i}{\sqrt{2}} (a^\dagger - a). \quad (6)$$

Upon scaling the Hamiltonian by  $\hbar\omega_1$  we obtain the dimensionless form,

$$H(t) = a^\dagger a + \frac{1}{2} + \beta \left( \frac{a^\dagger + a}{\sqrt{2}} \right)^4 + f \cos(\omega_{ex}t) \left( \frac{a^\dagger + a}{\sqrt{2}} \right), \quad (7)$$

with the redefined dimensionless parameters,

$$\beta = \frac{\tilde{\beta} x_0^4}{4\hbar\omega_1}, \quad f = \frac{\tilde{f} x_0}{\hbar\omega_1}, \quad (8)$$

where  $x_0 = \sqrt{\hbar/m^*\omega_1}$ . One can obtain an expression for  $\beta$  that depends on the dimensions and the material properties of the beam. By substituting the parameters in Eq. (8) one finds

$$\beta = \frac{\hbar}{2} \frac{1}{L w d^2} \frac{1}{\sqrt{\rho E}} \frac{1 + 3\epsilon}{\epsilon^{3/2}}. \quad (9)$$

Note that the equation (9) is valid for  $\epsilon \ll 1$  and  $\beta$  can be controlled by fine tuning the distance parameter  $\epsilon$  at this regime. Table I lists the range of the nonlinearity parameter  $\beta$  as well as the relaxed fundamental frequencies and the critical compressions for three different sizes of Si nanobeams. Note that one should have extremely precise control over  $\epsilon$  to increase the nonlinearity. In light of the measurements done in the experiment [23]  $\epsilon$  was found to be of the size  $\approx 10^{-5}$  for a 100 nm length carbon nanotube.

TABLE I: Table shows calculated  $\beta$  values, the nonlinearity parameter, for two different beam size. For each size, the relaxed fundamental mode frequency  $\omega_0$  and the critical force  $F_c$  defined in the text, are also shown. The parameter  $\epsilon$  shows the distance to the critical force near the Euler instability.

Si bar, $\rho = 2330 \text{ kg/m}^3$ , $E = 169 \text{ GPa}$		
$L = 200 \text{ nm}$	$\omega_0/2\pi = 1.12 \text{ GHz}$	$\epsilon = 10^{-6} \implies \beta = 0.05$
$d = 5 \text{ nm}$	$F_c = 4.34 \text{ nN}$	$\epsilon = 10^{-7} \implies \beta = 1.68$
$w = 10 \text{ nm}$		
$L = 400 \text{ nm}$	$\omega_0/2\pi = 557 \text{ MHz}$	$\epsilon = 10^{-7} \implies \beta = 0.11$
$d = 10 \text{ nm}$	$F_c = 17.3 \text{ nN}$	
$w = 20 \text{ nm}$		

### III. PREVIOUS WORKS ON THE SQUEEZING IN NONLINEAR OSCILLATORS

The squeezing produced by the nonlinearity has been investigated in the past by using a number of approximations however none of these are suitable for the problem of the NAMO. Milburn dropped all phase sensitive terms from (7) and studied [24] the simplified Hamiltonian,

$$H = \hbar\omega[(1 + \beta)a^\dagger a + \beta(a^\dagger a)^2]. \quad (10)$$

By solving exactly the phase space distribution function he showed that squeezing can be obtained for a coherent state of amplitude  $\alpha = 0.5$  for very short times. Buzek [25] and Tanas [26] studied Hamiltonian models of the form,

$$H = \hbar\omega a^\dagger a + \frac{1}{2} \hbar\beta a^{\dagger 2} a^2, \quad (11)$$

$$H = \hbar\omega a^\dagger a + \frac{1}{2} \hbar\beta (a^\dagger a)^2. \quad (12)$$

They solved the Heisenberg equations of motion exactly and showed periodic squeezing for coherent states in both quadratures. Tanas [26] showed also that maximum squeezing can be obtained in the limit of large mean number ( $|\alpha|^2 \gg 1$ ) and small times ( $t \ll 1$ ). No squeezing is allowed for the vacuum (ground) state in the above models.

In this paper, we calculate the squeezing in an anharmonic oscillator for the anharmonicity quartic in  $x$  which is the amplitude of the fundamental mode of the oscillation. One could write  $x^4$  in terms of creation and annihilation operators as follows:

$$x^4 \propto (a^\dagger + a)^4 = a^{\dagger 4} + a^4 + f(a^\dagger, a) \quad (13)$$

where  $f(a^\dagger, a)$  is a polynomial in  $a$  and  $a^\dagger$  of order four which is given by

$$f(a^\dagger, a) = 6 a^{\dagger 2} a^2 + 4(a^{\dagger 2} a^\dagger a + a^\dagger a a^2) + 6(a^{\dagger 2} + a^2) + 12a^\dagger a + 3. \quad (14)$$

The Hamiltonian containing the second and third terms given in Eq. (13) give rise to two-photon (or phonons in quantum mechanical descriptions of solid systems) transitions. It is known that two-photon transitions are necessary for producing squeezed states and thus the terms  $a^\dagger a a^2$ ,  $a^2$  and their hermitian conjugate should be important. In the literature, multi-photon processes have also been analyzed to study normal and higher order squeezing in the limit of small times [27, 28, 29]. In relation to this paper, Tombesi and Mecozzi [30] studied the harmonic oscillator model which has four-photon transitions in the interaction term,

$$H_I = \beta[a^{\dagger 4} + a^4 - (a^{\dagger 2} a^2 + a^2 a^{\dagger 2})]. \quad (15)$$

This model was solved exactly. They showed that significant amount of normal and higher order squeezing is possible for initial coherent states of amplitude  $|\alpha| > 1$  with certain phases and for short times. No squeezing is allowed for the vacuum and fluctuations diverge as time grows.

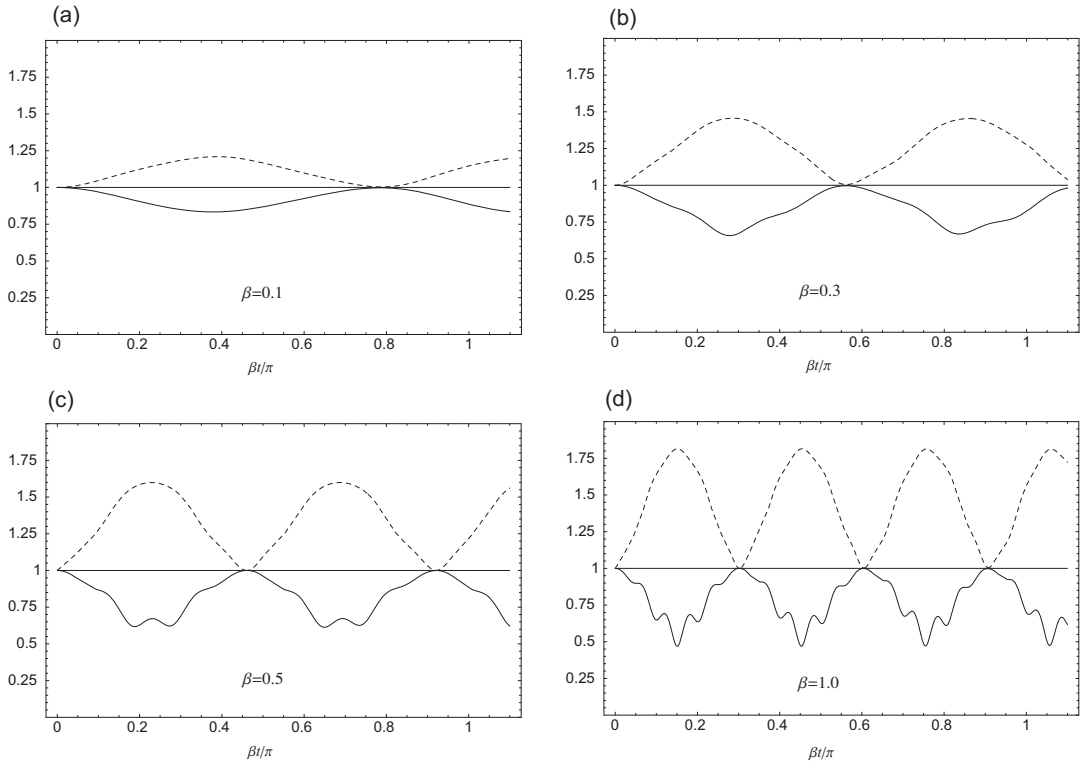


FIG. 2: Squeezing of the ground state in the anharmonic oscillator model given by Eq. (7) changing from weak to strong nonlinearity  $\beta$ ; (a)  $\beta = 0.1$ , (b)  $\beta = 0.3$ , (c)  $\beta = 0.5$ , (d)  $\beta = 1.0$ . Solid line,  $S_x(t)$ , normally ordered normalized fluctuations in  $x$  and dashed line,  $S_p(t)$ , normally ordered normalized fluctuations in  $p$  as given by Eqs. (18) and (19) respectively.

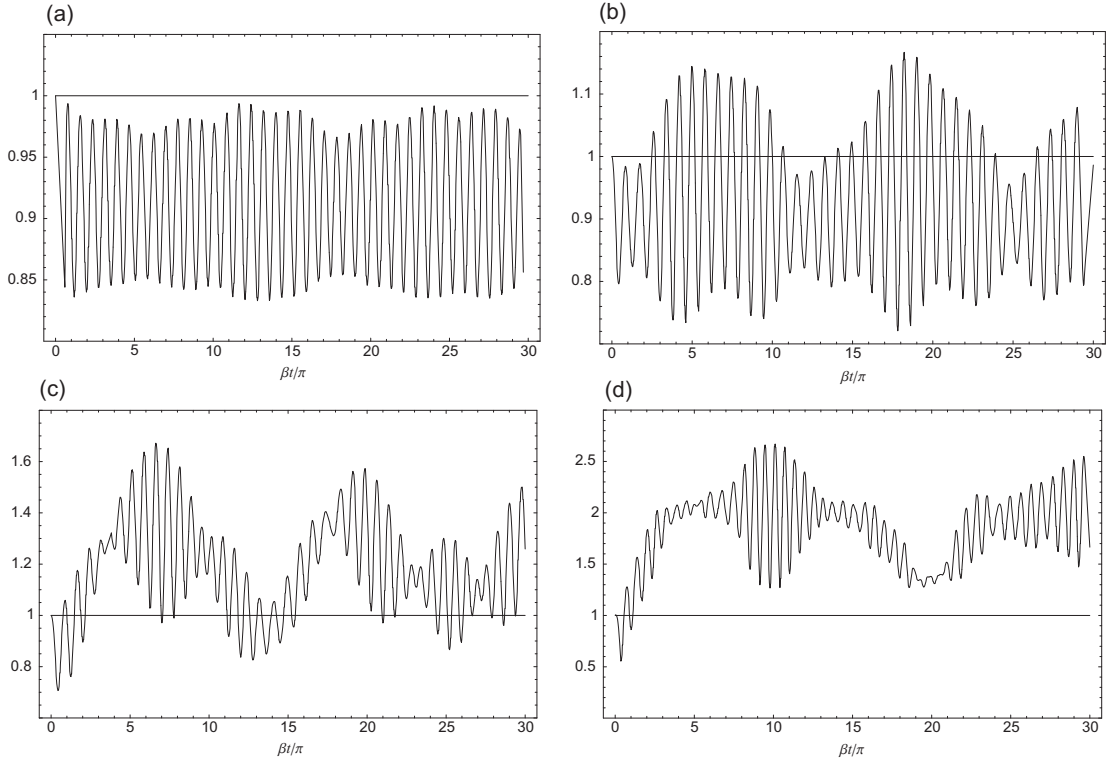


FIG. 3: The squeezing  $S_x(t)$  for the coherent state of amplitudes, (a)  $|\alpha| = 0.1$ , (b)  $|\alpha| = 0.5$ , (c)  $|\alpha| = 1.0$ , (d)  $|\alpha| = 2.0$ . The phase of  $\alpha$  is equal to  $\pi/2$ . The nonlinearity parameter  $\beta$  is taken as 0.1.

In the description of NMO systems the anharmonic Hamiltonian models given in Eqs. (10), (11), (12) and (14) are no good for vacuum (ground state) squeezing. Moreover, the nonlinearity  $\lambda$  cannot be controlled externally since it is an intrinsic property of the medium. To observe the nonclassical (quantum) properties of a mesoscopic system in general, the control of the parameter that gives rise to the nonclassical behavior would be crucial for an experimentalist.

The harmonic oscillator having the nonlinearity of Eq. (13) that we shall work in the next sections, gives important squeezing in the in-phase quadrature for both ground state and coherent states. Furthermore, it will be shown in section III that the physical model of the NMO allows one to control the nonlinearity by the application of a static external force. The numerical solution of the Hamiltonian shows that the ground state squeezing displays periodicity and it stays squeezed for the whole cycle of the period. In fact, the ground state squeezing is important for the mesoscopic resonators because bringing the harmonic oscillator representing the nanomechanical system to its vibrational ground state is a necessary prerequisite for quantum state engineering. The effect of driving term is also examined.

#### IV. QUANTUM DYNAMICS AND SQUEEZING

We first consider the case in which there is no driving. The Hamiltonian

$$H = a^\dagger a + \frac{1}{2} + \beta \left( \frac{a^\dagger + a}{\sqrt{2}} \right)^4, \quad (16)$$

has no analytical solution. For the numerical calculations, we employ the split-operator method [31] for the time propagation of the initial state. In this method, one can split the propagator on a time step  $\Delta t$  as

$$\begin{aligned} U(t + \Delta t) &= e^{-i(H_0 + V)\Delta t} \\ &= e^{-iH_0\Delta t/2} e^{-iV\Delta t} e^{-iH_0\Delta t/2} + O[(\Delta t)^3], \end{aligned} \quad (17)$$

where  $H_0 = a^\dagger a + 1/2$  and  $V = \beta(a^\dagger + a)^4/4$ . That means splitting the exponential of the operators which are not commuting is accurate to second order in the time step  $\Delta t$ . Therefore one can make the calculation as accurate as possible by taking the time step sufficiently small. Now we are ready to analyze the dynamics of the oscillations of the fundamental mode. We take the ground state of the Hamiltonian  $H_0$  as our initial state. Then we analyze the dynamics by the application of compressive force  $F_0$  suddenly (in a time interval much faster than the oscillator's frequency) in the regime near to the Euler instability where the nonlinearity parameter  $\beta$  can go from negligibly small values (where the dynamics is largely governed by ) to appreciable values. One can see how the nonlinearity parameter  $\beta$  changes by tuning the distance parameter  $\epsilon$  in the table I. We examine the dynamics by calculating the time evolution of the

width of initial wave function in the “coordinate”-space, i.e.  $x$ -space (we take the amplitude of the transverse deflection of the NMO as our coordinate as explained in section II.) and the corresponding momentum-space, i.e.  $p$ -space. Next, we define the squeezing factors

$$S_x(t) = \frac{\Delta x(t)}{\Delta x_0}, \quad (18)$$

$$S_p(t) = \frac{\Delta p(t)}{\Delta p_0}, \quad (19)$$

where  $\Delta x(t) = (\langle x^2(t) \rangle - \langle x(t) \rangle^2)^{1/2}$ , and  $\Delta p(t) = (\langle p^2(t) \rangle - \langle p(t) \rangle^2)^{1/2}$ .  $\Delta x_0$  and  $\Delta p_0$  are the widths in coordinate and momentum space of the initial wave function. The squeezing occurs when one of the expressions in Eqs. 18 and 19 get less than one and it is said to be perfect when it gets zero. Fig. 2 shows squeezing for the ground state for different nonlinearity values  $\beta = 0.1, 0.3, 0.5$  and 1.0.

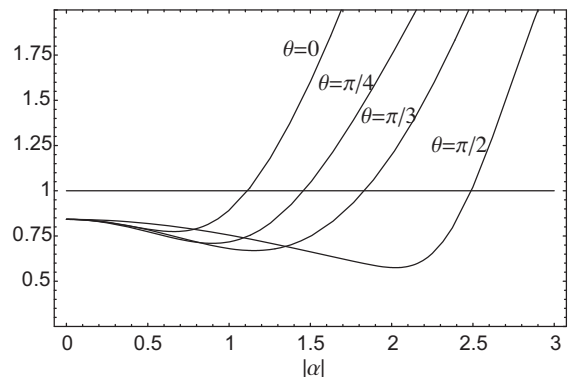


FIG. 4: Maximum squeezing in  $x$  vs. coherent state amplitude  $|\alpha|$ . The dependence is shown for different values of the phase. The nonlinearity parameter  $\beta$  is equal to 0.1.

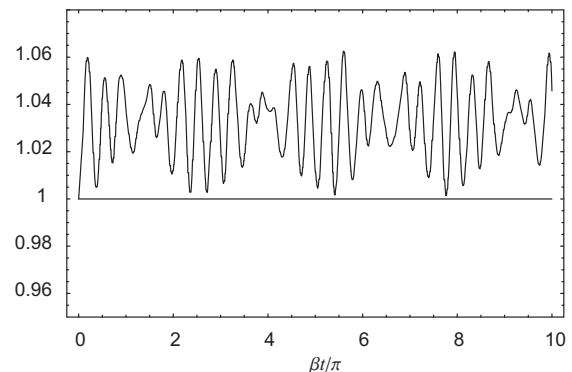


FIG. 5: Time evolution of the uncertainty product  $\Delta x(t)\Delta p(t)/\Delta x_0\Delta p_0$  for the ground state wave function of  $H_0$  under the dynamics of the Hamiltonian  $H$  given in Eq. (16).

One could also start with a coherent state to check squeezing. To prepare the coherent state, just as in

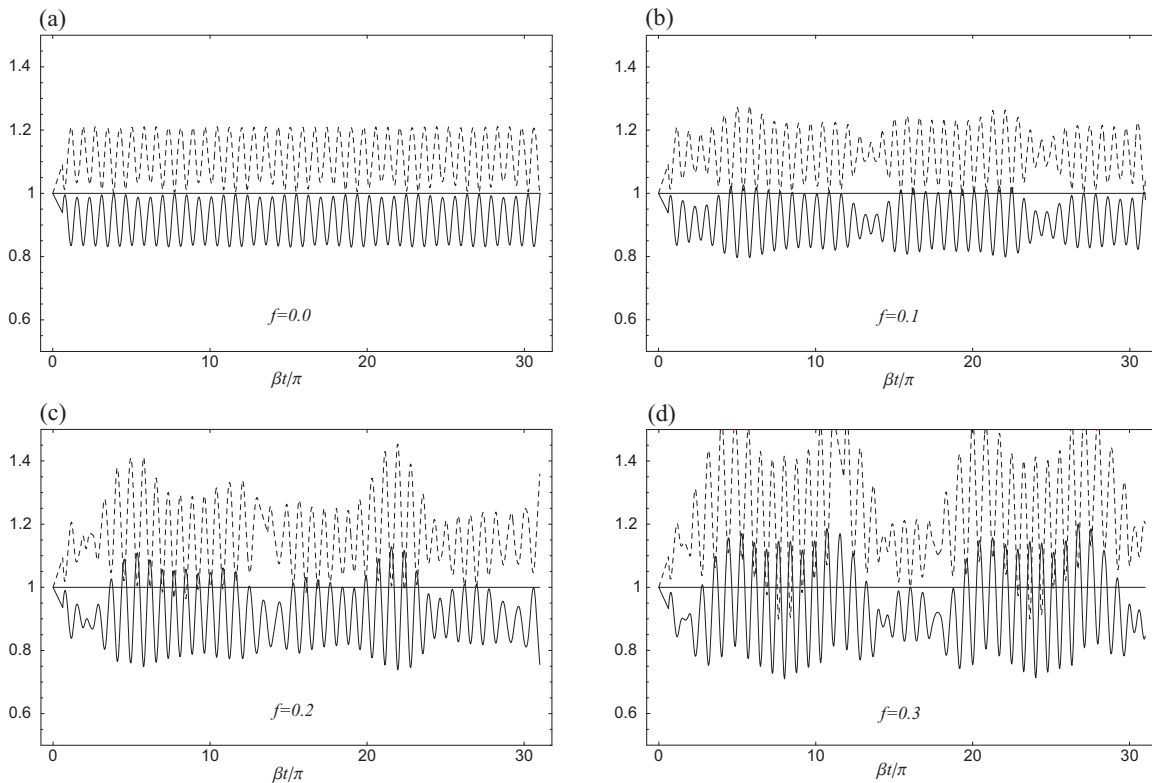


FIG. 6: The effect of the driving term on squeezing. (a) shows the squeezing without driving and (b), (c), (d) show the effect for the dimensionless driving parameter ( $f$ ) values of 0.1, 0.2 and 0.3 respectively. The nonlinearity parameter  $\beta$  is equal to 0.1. Solid line is for  $S_x(t)$  and the dashed line is for  $S_p(t)$ .

the preparation of ground state mentioned in the previous paragraph, one can first start with the linear harmonic oscillator Hamiltonian  $H_0$ . Then the coherent state can be obtained by resonantly driving (e.g. by a harmonic electromotive force) [32] the systems ground state. Thereafter, by switching the nonlinearity suddenly the dynamics of squeezing can be obtained. We analyzed squeezing for different initial coherent state amplitudes and phases,  $\alpha = |\alpha|e^{i\theta}$ , at the nonlinearity value of  $\beta = 0.1$ . Fig. 3 shows the time evolution of the squeezing for increasing coherent state amplitude  $\alpha$  at the phase  $\theta = \pi/2$ . Fig. 4 shows the dependence of the maximum squeezing to the phase. It can be seen from the figure that the phase does not make much difference for low amplitudes ( $|\alpha| < 0.5$ ) but it changes squeezing behavior drastically for amplitudes larger than 0.5. Increasing the amplitude increases the maximum squeezing whereas the state never becomes squeezed after a short duration. On the other hand, low amplitudes shows periodic squeezing all the time at a moderate value.

One can also plot the time evolution of the uncertainty product  $\Delta x(t)\Delta p(t)/\Delta x_0\Delta p_0$  for the ground state wave function of the linear Hamiltonian  $H_0$ . Fig. 5 shows that the oscillator recovers its minimum uncertainty periodically and the fluctuation remains bounded.

Next we analyze the effect of driving in the dynamics of the Hamiltonian given in Eq. (7). We calculate

the propagator again by using the split-operator method. This time, we include the nonlinear term into  $H_0$  and we take the time dependent driving term as  $V(t)$  to employ the splitting given in Eq. (17). Fig. 6 shows the time evolution of the squeezing factors given in Eqs. (18) and (19) for the in-phase and the out-of-phase quadratures for driving parameter values of  $f = 0.0, 0.1, 0.2$  and  $0.3$  at the nonlinearity  $\beta = 0.1$ . We take the ground state of the linear Hamiltonian as the initial state. The frequency of the driving term,  $\omega_{ex}$ , is on resonance with the frequency of the oscillator,  $\omega_1$ . As clearly seen, the effect of the driving is to enhance the squeezing in  $x$  periodically to a larger value. For the values of  $f$  in consideration, maximum squeezing goes from 17% (for  $f = 0.0$ ) to 30% (for  $f = 0.3$ ). Increasing the driving further does not enhance squeezing, rather it starts to enhance the fluctuations and the squeezing cycle becomes shorter. And the driving frequencies off the resonance does not help enhancing the squeezing at all. The upper limit  $\bar{f}_{max}$  for the regime of validity of the fundamental mode description is given by the first harmonic threshold, that is  $\bar{f}_{max} < \hbar\omega_2 \approx 3\hbar\omega_1$  [13] which means for the dimensionless parameter,  $f_{max} < 3$ .

We include the damping effects in the master equation formalism of the form

$$\dot{\rho} = -i[H, \rho] + \gamma(a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a), \quad (20)$$

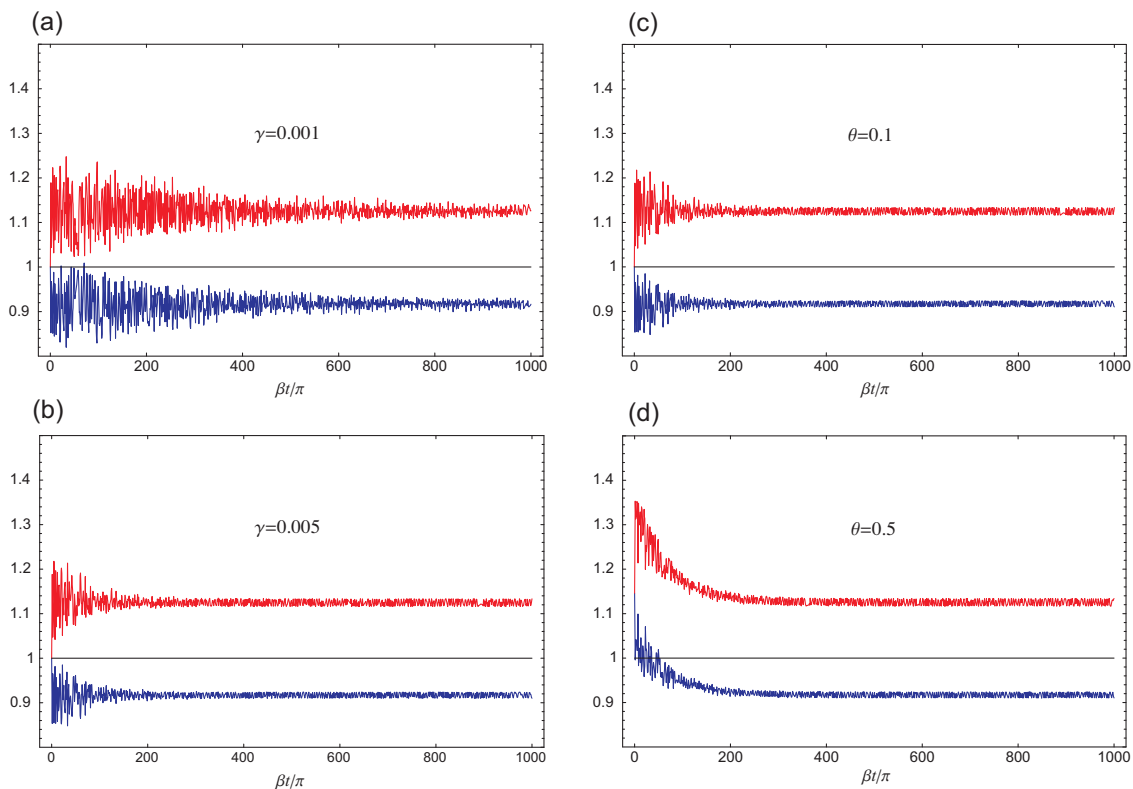


FIG. 7: (Color online) Squeezing in the driven damped anharmonic nanomechanical oscillator. The red and blue lines show  $S_p(t)$  and  $S_x(t)$  respectively. (a) and (b) show squeezing for the ground state with the damping constants  $\gamma = 0.001\omega_1$  and  $\gamma = 0.005\omega_1$  respectively. (c) and (d) show squeezing for the thermal state with the damping constant  $\gamma = 0.005\omega_1$  at the temperatures  $kT = 0.1\hbar\omega_0$  and  $kT = 0.5\hbar\omega_0$  respectively. The other parameters are kept at  $\beta = 0.1$  and  $f = 0.1$  for all.

where  $H$  is the Hamiltonian given by the Eq. (7) and  $\gamma$  is the damping constant. Here,  $\gamma$  is the rate for the oscillator to decay from the first excited state to the ground state. The rate for the decay from level  $n$  to  $n - 1$  is  $n\gamma$ . After solving the master equation numerically we plot the squeezing factors  $S_x(t)$  and  $S_p(t)$  in Figs. 7(a) and 7(b) for the ground state. As seen clearly, the damping effect shows itself as reducing the fluctuations exponentially to a steady state value. The master equation formalism also allows us to analyze the squeezing for the thermal state of the relaxed beam (described by the linear Hamiltonian  $H_0$ ),  $\rho(t = 0) = e^{-H_0/\theta}/Z$ , where  $H_0 = a^\dagger a + 1/2$ ,  $\theta = kT/\hbar\omega_0$  and  $Z = \text{Tr}(e^{-H_0/\theta})$ . Figs. 7(c) and 7(d) show that one need not to start at 0 K to obtain squeezing. As long as one keeps the noise of the environment sufficiently low, at higher temperatures of the beam ( $kT$  of fractions of the level spacing  $\hbar\omega_0$ ), although there is no substantial squeezing initially, at steady state, the amount of “coordinate” squeezing is reaching to the ground state value.

## V. CONCLUSION

In this paper we show that squeezed states can be obtained in the amplitude of the fundamental mode of a

nanomechanical oscillator which has quartic nonlinearity in its effective single particle quantum mechanical Hamiltonian. The quantum dynamics of the system is solved numerically both with and without external ac-driving. For various strength of nonlinearity and driving, the squeezing dynamics is investigated for both initial ground state and coherent states of different amplitudes. The terms that lead to multilevel transitions in the quartic nonlinearity is compared to similar models and proved advantageous results. It should be noted that working closer to Euler buckling instability produces larger squeezing due to large nonlinearity. This is reminiscent of similar result for quantum optical systems [33].

Finally, we comment on the possibility of observing squeezing in nanomechanical beams. The obvious way to detect squeezing seems to be one of the displacement detection methods. There are different displacement sensing techniques such as optical, magnetomotive and single-electron transistor (SET) technique. Among these read-out strategies, the SET –demonstrated by many research groups to be a very sensitive detector of charge– enables position detection for NMO devices in their low energy quantum states. In this detection method, a charged NMO is capacitively coupled to an SET. The oscillator’s motion induces a change in the charge on the gate electrode of the SET and the change in the SET’s con-



ductance can be directly monitored. Demonstrations of position detection of NMO's are developing. The first experiment [34] reached the position sensitivity within a factor of 100 of the quantum limit  $\Delta x_{SQL} = \sqrt{\hbar/2m\omega_0}$ . A year after another group [4] improved this sensitivity to a factor of 4.3 of the quantum limit set by the Heisenberg uncertainty principle of a Nanomechanical resonator nearly its ground state. These demonstrations suggests that beating the standard quantum limit in the position detection technology is highly probable in future. Once this is achieved, by reading out the position each time with a different nonlinearity parameter one can prove the existence of tunable squeezing. There are also proposals suggesting the positioning of a NMO inside an electromagnetic cavity. It is proposed that the interaction of cavity photons with a NMO in a squeezed state would

produce a nonclassical statistics of photons at the exit. So one can easily detect squeezing simply by quantum statistical analysis of photons at the output of the cavity.

In conclusion, in nanomechanical beams described in this paper, one can remarkably control the quantum squeezing externally just by controlling the strain provided by a classical compressive force. However, to control the nonlinearity at the desired values, one would have to apply compression with extreme delicacy,  $F_c - F_0 \sim 10^{-6}F_c$ . Controlling the strain to this precision for sufficient time to identify squeezing may be difficult. Thus, while observing squeezing will be challenging, the prospect of exploring tunable quantum squeezing in nanomechanical beams and the connection to Euler buckling instability are intriguing.

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